

SPIN ENTANGLEMENT AND NONLOCALITY OF MULTIFERMION SYSTEMS.

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The fundamental property of electrons – spin $S = 1/2$ – is very perspective for usage as the information carrier in spintronics, quantum computing and quantum cryptography as well as in any combination of these sciences. However, to be used any electron spin should be extracted from the ensemble of undistinguished particles, and spin properties of such ensemble should be known.

Spin states of odd number N of undistinguished fermions and their properties have been studied in details. Spin states are proved to be determined by the Pauli's principle unequivocally and should be described by spin density matrices (SDM) [2]. The initial step for calculation of SDM is the antisymmetric fermion wavefunction chosen as the Slater determinant. Such SDM are shown can be presented as the normalized sum of nonorthogonal projector operators onto all possible multifermion singlet states

$$\rho_N = 2^{N/2} (N!)^{-1} (N/2)! \sum_P P(|S_{ij}S_{rl}S_{mn} \dots\rangle \langle S_{ij}S_{rl}S_{mn} \dots|)$$

Here P means all possible permutations of the indexes i, j, k, l, m, n, \dots , and $|S_{ij}\rangle = 2^{-1/2} |\uparrow_i \downarrow_j - \downarrow_i \uparrow_j\rangle$ are the singlet spin state of the fermion pair. The spin state of the simplest nontrivial four fermion system is

$$\rho_4 = 3^{-1} (|S_{12}S_{34}\rangle \langle S_{12}S_{34}| + |S_{13}S_{24}\rangle \langle S_{13}S_{24}| + |S_{14}S_{23}\rangle \langle S_{14}S_{23}|).$$

If the orthogonal spin basis will be used instead of the nonorthogonal one, then the SDM will take the form of normalized unity matrix that describes the noncoherent superposition of all the possible multifermion singlet spin states. The obtained SDM were used for investigations of spin correlations in multifermion systems.

The SDM of spin subsystems of the full systems can be calculated as the trace over spin states of fermions extracted from the initial systems. Results of such calculations prove that this algorithm is equivalent to usage of the multi determinant wavefunction of fermions.

The fact that the spin states of the four fermion system described by the spin density matrix ρ_4 are maximally entangled is proved by direct calculation of eigenvalues in accordance with the the Peres-Horodecki criterion. The Sylvester criterion of the matrix nonnegativity was used to prove that the partly transposed matrix has negative eigenvalues, and multifermion spin states of the full system are entangled. Therefore, spin states of any N_1 -fermion subsystem ($N_1 = 1, 2, 3 \dots N_1 < N$) are entangled with spin states of the rest of the full system.

Violation of the Bell's inequalities was proved for the case if the initial fermion system decays and produces one fermion. Applications of the multispin entanglement to problems of quantum biology and state selective processes will be presented.